31. (b) We have $\sqrt{2} + \sqrt{8} + \sqrt{18} + \sqrt{32} + \ldots\ldots$

$= 1\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots\ldots\ldots$

$= \sqrt{2}[1 + 2 + 3 + 4 + \ldots\ldots\ldots\text{upto 24 terms}]$

32. (d) Given that $S_n = nA + n^2 B$

Putting $n = 1, 2, 3, \ldots\ldots\ldots$, we get

$S_1 = A + B, S_2 = 2A + 4B, S_3 = 3A + 9B$

....

....

Therefore $T_1 = S_1 = A + B, T_2 = S_2 - S_1 = A + 3B,$

$T_3 = S_3 - S_2 = A + 5B$,

....

....

Hence the sequence is $(A + B),(A + 3B), (A + 5B),\ldots\ldots$

Here $a = A + B$ and common difference $d = 2B$.

33. (d) $\log_3 2, \log_3 (2^x - 5)$ and $\log_3 \left(2^x - \dfrac{7}{2}\right)$ are in A.P.

$\Rightarrow 2\log_3 (2^x - 5) = \log_3\left[(2)\left(2^x - \dfrac{7}{2}\right)\right]$

$\Rightarrow (2^x - 5)^2 = 2^{x+1} - 7 \Rightarrow 2^{2x} - 12 \cdot 2^x - 32 = 0$

$\Rightarrow x = 2, 3$

But $x = 2$ does not hold, hence $x = 3$.

34. (a) $p\{a + (p - 1)d\} = q\{a + (q - 1)d\}$

$\Rightarrow a(p - q) + (p^2 - q^2)d + (q - p)d = 0$

$\Rightarrow (p - q)\{a + (p + q - 1)d\} = 0$

$\Rightarrow a + (p + q - 1)d = 0 \Rightarrow T_{p+q} = 0$, $\{\because p \neq q\}$.

35. (c) $T_m = a + (m - 1)d = \dfrac{1}{n}$

and $T_n = a + (n - 1)d = \dfrac{1}{m}$

On solving $a = \dfrac{1}{mn}$ and $d = \dfrac{1}{mn}$

$\therefore T_{mn} = a + (mn - 1)d = \dfrac{1}{mn} + (mn - 1)\dfrac{1}{mn} = 1$

36. (c) Let S_n and S'_n be the sums of n terms of two A.P.'s and T_{11} and T'_{11} be the respective 11th terms, then

$\dfrac{S_n}{S'_n} = \dfrac{\dfrac{n}{2}[2a + (n - 1)d]}{\dfrac{n}{2}[2a' + (n - 1)d']} = \dfrac{7n + 1}{4n + 27}$

$\Rightarrow \dfrac{a + \dfrac{(n - 1)}{2}d}{a' + \dfrac{(n - 1)}{2}d'} = \dfrac{7n + 1}{4n + 27}$

Now put $n = 21$,

we get $\dfrac{a+10d}{a'+10d'} = \dfrac{T_{11}}{T'_{11}} = \dfrac{148}{111} = \dfrac{4}{3}$.

Note : If ratio of sum of n terms of two A.P.'s are given in terms of n and ratio of their p^{th} terms are to be found then put $n = 2p - 1$. Here we put $n = 11 \times 2 - 1 = 21$.

37. (c) Let $S_{Even} = 2 + 4 + 6 + 8 + \ldots\ldots\infty$ …..(i)

and $S_{Odd} = 1 + 3 + 5 + 7 + 9 + \ldots\ldots\infty$ …..(ii)

Sum $S_E = \dfrac{n}{2}[4 + (n-1)2] = \dfrac{n}{2}[2n+2] = \dfrac{n}{2}2(n+1)$

and $S_O = \dfrac{n}{2}[2 + (n-1)2] = \dfrac{n}{2}(2n)$

Now $\dfrac{S_E}{S_O} = \dfrac{(n+1)}{n}$ or $S_E : S_O = (n+1) : n$.

38. (a) n^{th} term of the series is $20 + (n-1)\left(-\dfrac{2}{3}\right)$.

For sum to be maximum, n^{th} term ≥ 0

$\Rightarrow 20 + (n-1)\left(-\dfrac{2}{3}\right) \geq 0 \Rightarrow n \leq 31$

Thus the sum of 31 terms is maximum and is equal to

$\dfrac{31}{2}\left[40 + 30 \times \left(-\dfrac{2}{3}\right)\right] = 310$.

39. (a) We have $(x+1) + (x+4) + \ldots\ldots + (x+28) = 155$

Let n be the number of terms in the A.P. on L.H.S. Then $x + 28 = (x+1) + (n-1)3 \Rightarrow n = 10$

$\therefore (x+1) + (x+4) + \ldots\ldots + (x+28) = 155$

$\Rightarrow \dfrac{10}{2}[(x+1) + (x+28)] = 155 \Rightarrow x = 1$.

40. (b) $T_5 = ar^4 = \dfrac{1}{3}$ …..(i)

and $T_9 = ar^8 = \dfrac{16}{243}$ …..(ii)

Solving (i) and (ii), we get $r = \dfrac{2}{3}$ and $a = \dfrac{27}{16}$

Now 4th term $= ar^3 = \dfrac{3^3}{2^4} \cdot \dfrac{2^3}{3^3} = \dfrac{1}{2}$.

41. (a) Series is a G.P. with $a = 0.9 = \dfrac{9}{10}$ and $r = \dfrac{1}{10} = 0.1$

$\therefore S_{100} = a\left(\dfrac{1-r^{100}}{1-r}\right) = \dfrac{9}{10}\left(\dfrac{1 - \dfrac{1}{10^{100}}}{1 - \dfrac{1}{10}}\right) = 1 - \dfrac{1}{10^{100}}$.

42. (b) Given series $6 + 66 + 666 + \ldots\ldots +$ upto n terms

$= \dfrac{6}{9}(9 + 99 + 999 + \ldots\ldots$ upto n terms$)$

$= \dfrac{2}{3}(10 + 10^2 + 10^3 + \ldots\ldots +$ upto n terms $-n)$

$= \dfrac{2}{3}\left(\dfrac{10(10^n - 1)}{10 - 1} - n\right) = \dfrac{1}{27}[20(10^n - 1) - 18n]$

$= \dfrac{2(10^{n+1} - 9n - 10)}{27}$.

43. (b) Let G_1, G_2, G_3, G_4, G_5 be the G.M.'s are inserted between 486 and 2/3. So total terms are 7.

$T_n = ar^{n-1} \Rightarrow 2/3 = 486(r)^6 \Rightarrow r = 1/3$

Hence 4th G.M. will be, $T_5 = ar^4 = 486(1/3)^4 = 6$.

44. (a) Since the series are G.P., therefore

$$x = \frac{1}{1-a} \Rightarrow a = \frac{x-1}{x} \text{ and } y = \frac{1}{1-b} \Rightarrow b = \frac{y-1}{y}$$

$$\therefore 1 + ab + a^2b^2 + \ldots \ldots \infty = \frac{1}{1-ab}$$

$$= \frac{1}{1 - \dfrac{x-1}{x} \cdot \dfrac{y-1}{y}} = \frac{xy}{x+y-1}.$$

45. (a) $4^{1/3} \cdot 4^{1/9} \cdot 4^{1/27} \ldots \ldots \infty$

$$\therefore S = 4^{1/3 + 1/9 + 1/27} \ldots \ldots \infty$$

$$\Rightarrow S = 4^{\left(\frac{1/3}{1-1/3}\right)} = 4^{\frac{1/3}{2/3}} \Rightarrow S = 4^{1/2} \Rightarrow S = 2.$$